Find the average value of the function on the given interval.

1) $f(x)=x^2, \quad [-1, 1]$ $\boxed{\frac{1}{3}}$

2) $g(x)=x^2\sqrt{1+x^3}, \quad [0, 2]$ $\boxed{\frac{26}{9}}$

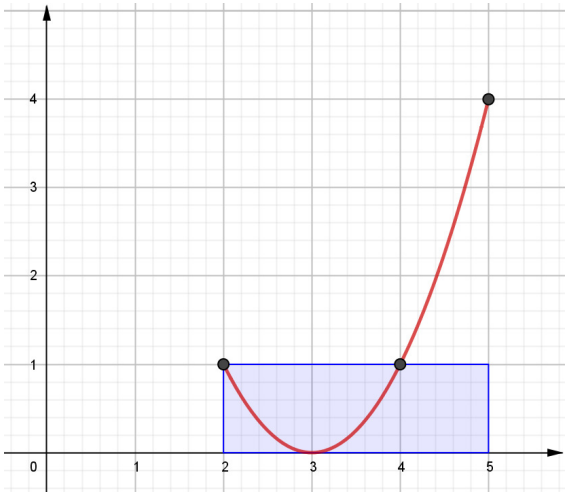
3) $f(t)=te^{-t^2}, \quad [0, 5]$ $\boxed{\frac{1}{10}(1-e^{-25})}$

4) $h(x)=\cos^4 x \sin x, \quad [0, \pi]$ $\boxed{\frac{2}{5\pi}}$

Find the average value of $f$ on the given interval. Find $c$ such that $f_{ave} = f(c)$. Sketch the graph $f$ and a rectangle whose area is the same as the area under the graph of $f$. You may need to use a graphing calculator.

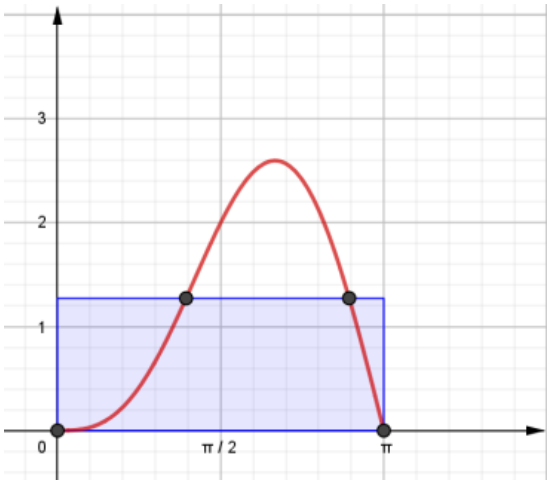
5) $f(x) = (x-3)^2, \quad [2, 5]$

$f_{ave} = 1$
$c = 2$ or $4$




6) $f(x) = 2\sin x - \sin 2x, \quad [0, \pi]$

$f_{ave} = \dfrac{4}{\pi}$
$c_1 \approx 1.238$ or $c_2 \approx 2.808$



7) If $f$ is continuous and $\int_1^3 f(x)\,dx = 8$, show that $f$ takes on the value 4 at least once on the interval $[1, 3]$.

$f$ is continuous on $[1, 3]$, so by the Mean Value Theorem for Integrals there exist a number $c$ in $[1, 3]$ such that $\int_1^3 f(x)\,dx = f(c)(3-1)$ which simplifies to $8 = 2f(c)$ so there is a number $c$ such that $f(c) = \frac{8}{2} = 4$.

8) Find $b$ such that the average value of $f(x) = 2 + 6x - 3x^2$ on the interval $[0, b]$ is equal to 3.

$$b = \frac{3 \pm \sqrt{5}}{2}$$

9) In a certain city the temperature (in °F) $t$ hours after 9 A.M was modeled by the function:

$$T(t) = 50 + 14\sin\frac{\pi}{12}t$$

Find the average temperature during the period from 9 A.M. to 9 P.M.

$$T_{ave} \approx 59°F$$

10) The linear density in a rod 8 m long is $\frac{12}{\sqrt{x+1}}$ kg/m , where $x$ , is measured in meters from one end of the rod. Find the average density of the rod.

$$\boxed{\rho_{ave} = 6 \text{ kg/m}}$$

11) The velocity $v$ of blood that flows in a blood vessel with radius $R$ and length $l$ at a distance $r$ from the central axis is:

$$v(r) = \frac{P}{4\eta l}(R^2 - r^2)$$

where $P$ is the pressure difference between the ends of the vessel and $\eta$ is the viscosity of the blood. Find the average velocity (with respect to $r$) over the interval $0 \le r \le R$. Compare the average velocity with the maximum velocity.

$$\boxed{\begin{aligned} v_{ave} &= \frac{PR^2}{6\eta l} \\ v_{ave} &= \frac{2}{3} v_{\max} \end{aligned}}$$